GRASP Laboratory
General Robotics & Autonomous Systems and Processes

Mechatronic Modeling and Design with Applications in Robotics

Basic Model Elements

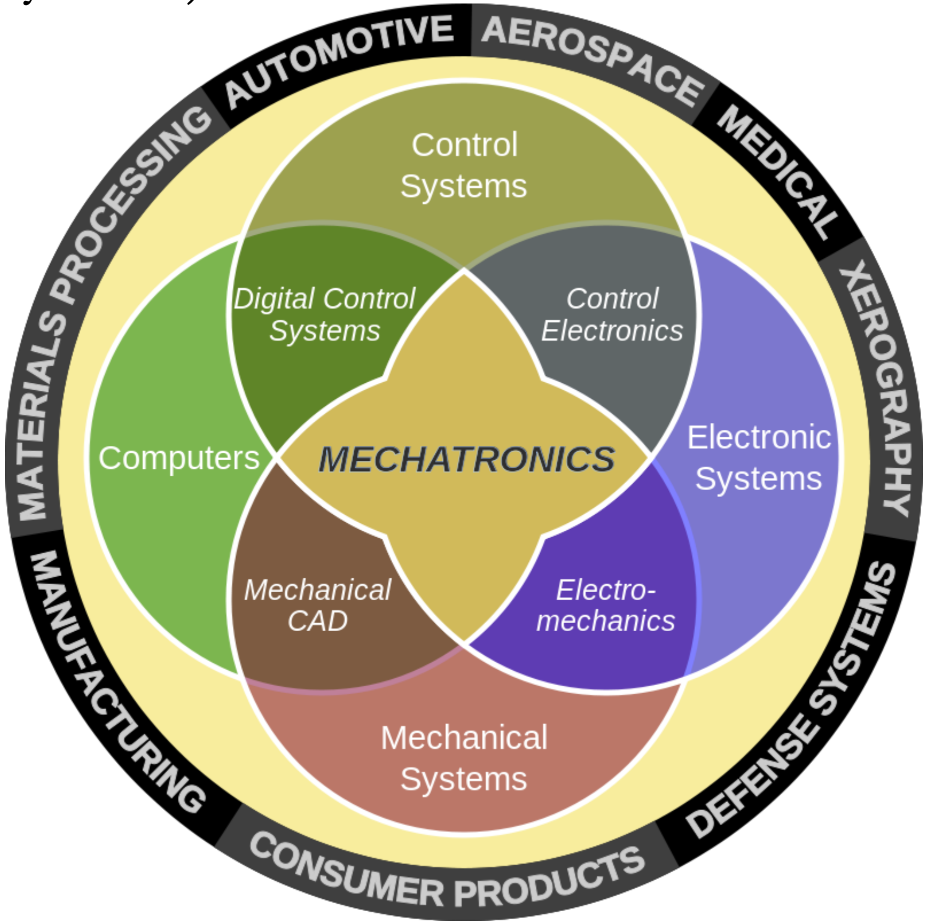
Mechatronic Systems

The field of mechatronics primarily concerns the integration of mechanics and electronics.
(e.g., mechanical, fluid, thermal and electrical/electronic systems)

They can serve functions of

- Structural support
- Load bearing
- Mobility
- Transmission of motion and energy
- Actuation
- Manipulation
- Sensing
- Control

Smart device.



https://en.wikipedia.org/wiki/Mechatronics

Electromechanical System

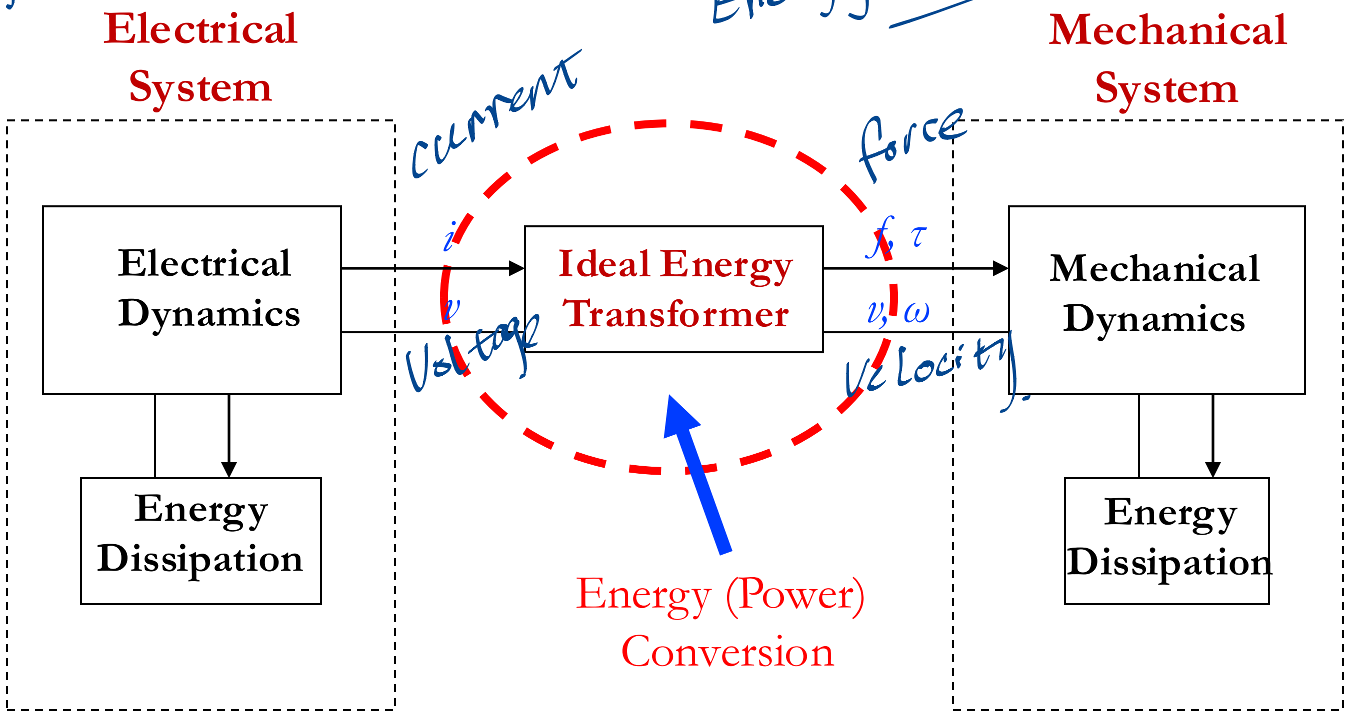
Mechatronics.

Similarity?

$f \times v = P$

Energy = Power × time.

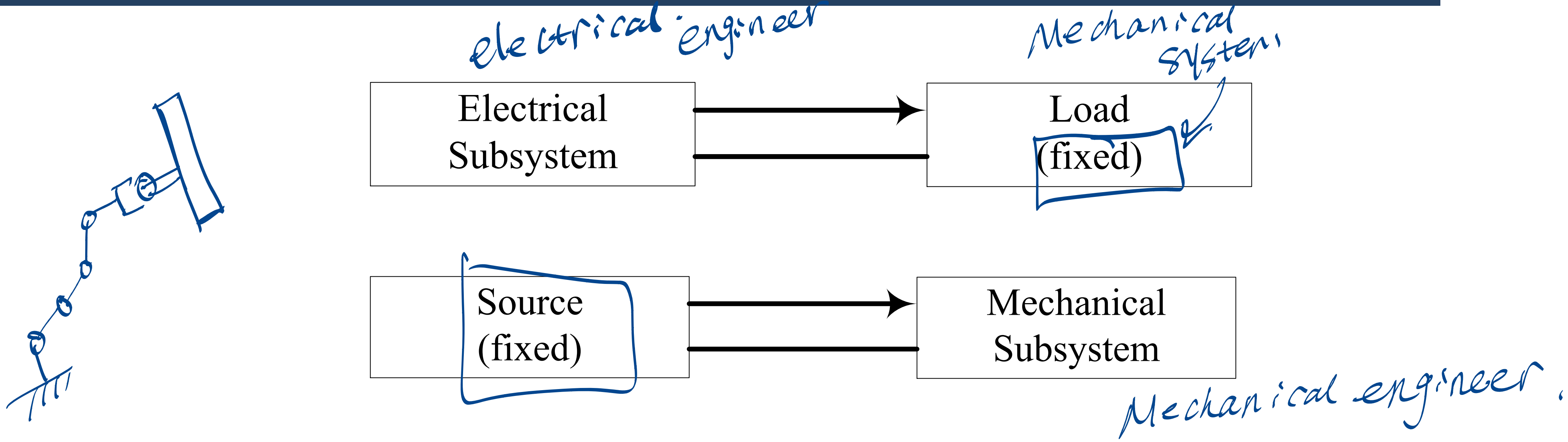
parameters:
Mechanical: f v
Electrical: i V
$i \times V = P$
power



An electromechanical system / mechatronic system

Mechatronic modeling / Design
(Energy method).

Distinction Between Mechanical and Electronic Components



- ❖ Energy (or Power)
- ❖ Bandwidth (e.g., Speed and Time Constant)

Basic Electrical Components

Required and needed in this course:

- Mechanical Components ✓
- Electrical Elements ✓

Should understand:

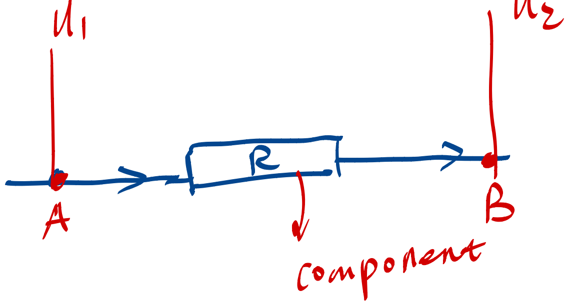
- ~~Fluid Elements~~
- ~~Thermal Elements~~

Reference.

Across and Through Variables

Variables: { Mechanical: f, v ; Electrical: i, V } 4 Variables.

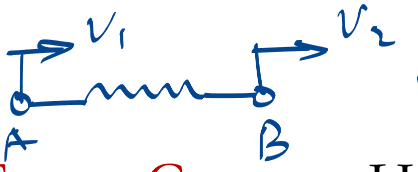
Across Variable: Varies Across Element (e.g., Velocity, Voltage, T~~emperature, Pressure~~)



$v = \Delta u = u_1 - u_2$.

$u_1 \neq u_2$.

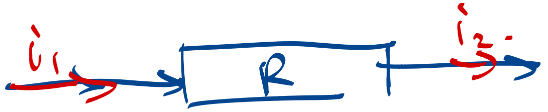
Voltage: V is a Across variable



$v_1 \neq v_2$

Through Variable: Remains Unchanged Through Element (e.g., Force, Current, ~~Heat Transfer Rate, Fluid Flow Rate~~)

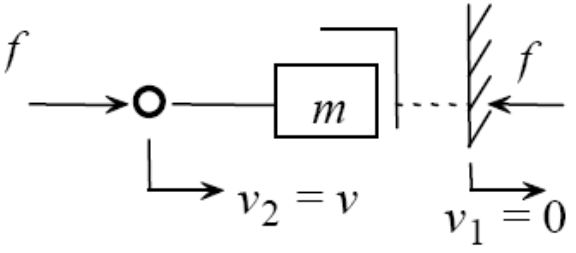
$i_1 = i_2$.



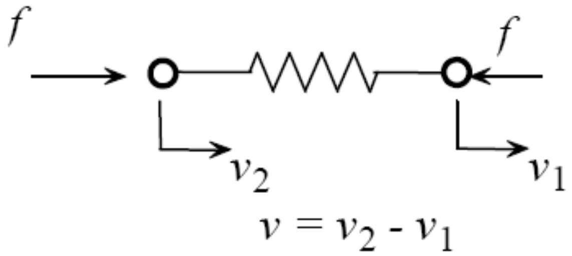
$|f_1| = |f_2|$

Mechanical Elements

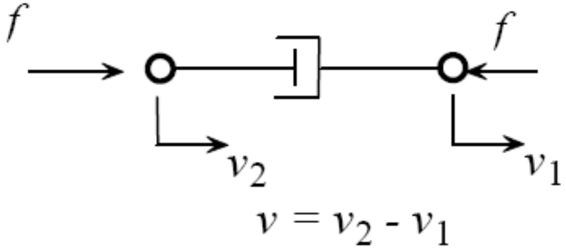
Mass



Spring



Damper



Mechanical source: force, velocity.

force → Through → T-Type source

velocity → Across → A-Type source

Sources: Velocity and force/torque

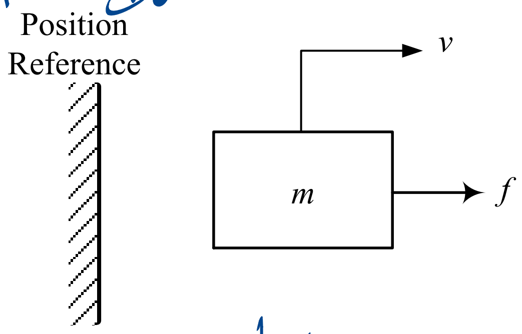
angular

Variables: Velocity (across variable) and force (through variable)

Source: T-Type source A-Type source.

Mechanical Element: Mass (Inertia)

Mass (Inertia) Element (A-Type Element)



Constitutive Equation (Newton's 2nd Law):

$$f = m\frac{dv}{dt}$$

where m = mass(inertia)

Power = $f\,v$ = rate of change of energy ➔

$$E = \int f v dt = \int m\frac{dv}{dt} v dt = \int m v dv$$

➔ Energy $E = \frac{1}{2}mv^2$ (Kinetic Energy) ➔ Energy storage element

$f = ma$ $a = \frac{f}{m} = \frac{dv}{dt}$.

$P = f \cdot V$

$P = f \cdot V = m\frac{dv}{dt} \cdot v$

$E = P \cdot dt$

$\int P dt = \int m \cdot v dv$

$E = \frac{1}{2} m v^2$ → Kinetic Energy. function of V.

$E = f(v) \Rightarrow f$?

V: velocity ⇒ Across variable

⇒ A-Type Element.

Observations: Mass (Inertia)

- An inertia is an energy storage element (kinetic energy).

- Velocity (across variable) represents the state of an inertia element ➔ "A-Type Element"

Note: 1. Velocity at any *t* is completely determined from initial velocity and the applied force; 2. Energy of inertia element is represented by *v* along.

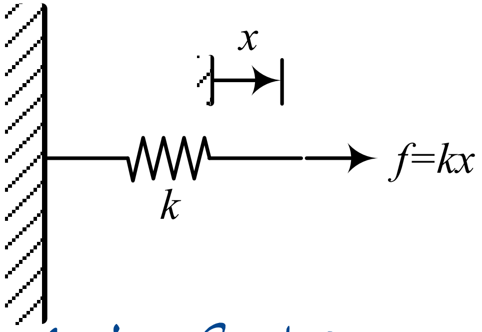
- Hence, *v* is a natural output (or response) variable for an inertia element, which can represent its dynamic state (i.e., state variable), and *f* is a natural input variable for an inertia element.

State-space model. → Mechanical system
select "v" → state variable of a mass

- Velocity across an inertia element cannot change instantaneously unless an infinite force is applied to it.

Mechanical Element: Spring (Stiffness)

Spring (Stiffness) Element (T-Type Element)



Constitutive Equation (Hooke's Law):

$$\frac{df}{dt} = kv$$

where *k*=stiffness

Note: Differentiated version of familiar force-deflection Hooke's law in order to use velocity (as for inertia element)

$$E = \int fvdt = \int f\frac{1}{k}df$$

➔ Energy $E = \frac{1}{2}\frac{f^2}{k}$ (Elastic potential energy)

➔ Energy storage element

$P = f \cdot v$

$v = \frac{1}{k}\frac{df}{dt}$

$P \cdot dt = E = \int f \cdot v \cdot dt = \int \frac{1}{k} \cdot f \cdot df$.

$E = \frac{1}{2}\frac{f^2}{k} \Rightarrow E = f(f)$

v ?

f: Through variable

Spring: T-Type element .

Observations: Spring (Stiffness)

- A spring (stiffness element) is an energy storage element (elastic potential energy).

- Force (through variable) represents state of spring element ➔ "T-Type Element".

Note: 1. Spring force of a spring at time *t* is completely determined from initial force and applied velocity; 2. Spring energy is represented by *f* alone.

- Force *f* is a natural output (response) variable, and *v* is a natural input variable for a stiffness element.

State-space model.

Choose "f" as state of spring.

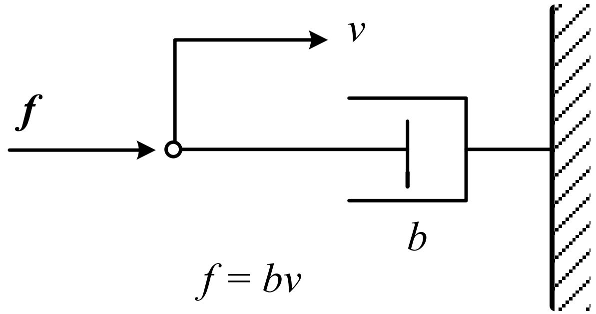
- Force through a stiffness element cannot change instantaneously unless an infinite velocity is applied to it.

Mechanical Element: Damping (Dissipation)

Damping (Dissipation) Element (D-Type Element)

Energy Dissipation element

D-Type Element



Constitutive Equation: $f = bv$

where b=damping constant (damping coefficient); for viscous damping

The power dissipated depending on the velocity v:

It dosen't store energy.

$$P = bv^2$$

Observations: Damping (Dissipation)

- Mechanical damper is an energy dissipating element (*D*-Type Element).

- Either force *f* or velocity *v* may represent its state.

- No new state variable is defined by this element.

Source T-Type : Force
A-Type : Velocity

T- Type
A - Type } element
D- Type

Spring
mass
damper

f : Through variable
V: Across Variable

Rotational Elements

Rotational Mass: A-Type element

$$E = \frac{1}{2} I \omega^2$$

Torsional Spring: T-Type element

$$E = \frac{1}{2} \frac{T^2}{k}$$

Rotary Damper: D-Type element

$$P = c\omega^2$$

$v \to \omega$

$f \to T$

$m \to I$.

ω : Across

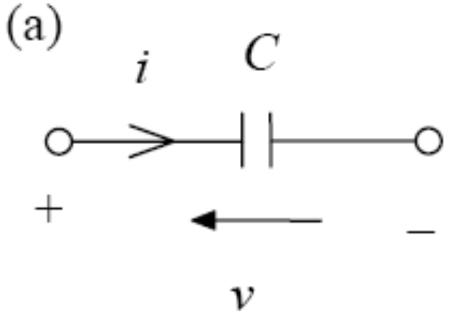
T : Through.

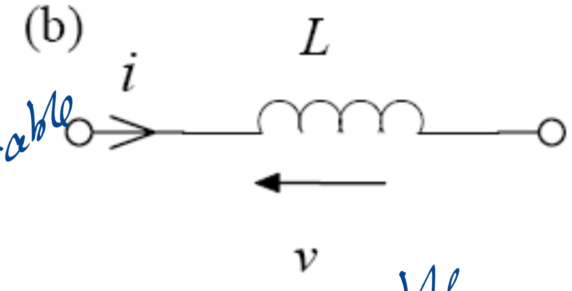
Electrical Elements

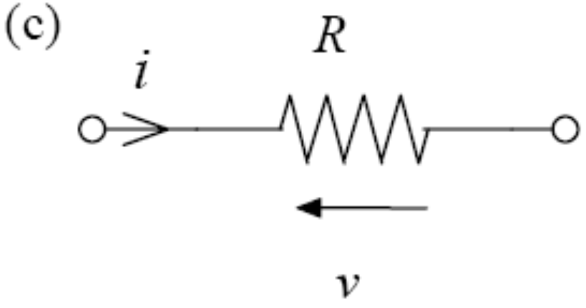
Capacitor A-Type element

Inductor → T-Type element

Resistor → D-Type element







Sources: Voltage and current

Across variable

Through variable.

Variables: Voltage (across variable) and current (through variable)

A-Type source

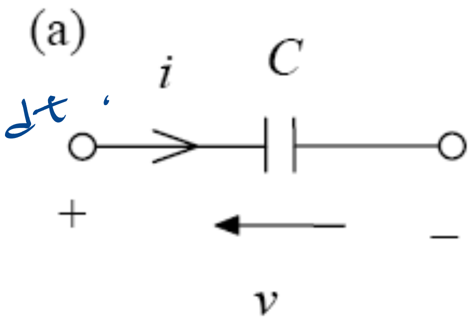
T-Type source.

Electrical Element: Capacitor

Variables: Voltage (across variable) and the current (through variable)

Capacitor Element (A-Type Element)

$P = v \cdot i$



Constitutive Equation: $C\frac{dv}{dt} = i$

$E = \int p \cdot dt = \int v \cdot i \cdot dt$

where C = capacitance

Power = iv ➔ Energy $E = \int ivdt = \int C\frac{dv}{dt}vdt = \int Cvdv$ ➔

Energy $E = \frac{1}{2}Cv^2$ (electrostatic energy) ➔ Energy storage element

$E = \int C(v)$ across variable

i ?

Capacitor: A-Type element.

State-Space: select "v" as the state of capacitor.

Observations: Capacitor

- Voltage (across variable) is state variable for a capacitor ➔ "A-Type Element".

- Voltage is a natural output variable and current is a natural input variable for a capacitor.

- Voltage across a capacitor cannot change instantaneously unless an infinite current is applied.

Electrical Element: Inductor

Inductor Element (T-Type Element)

Constitutive Equation: $L\frac{di}{dt} = v$

where L = inductance

Energy $E = \frac{1}{2}Li^2$ (Electromagnetic energy)

$\downarrow$ $f(i)$

$P = i \times v$

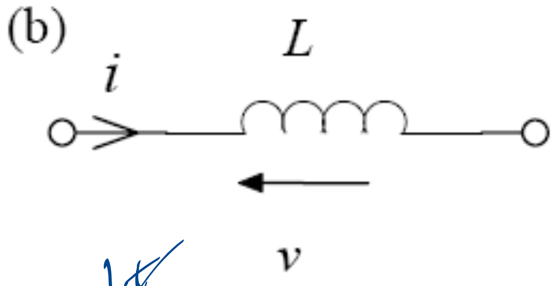
$E = \int p\,dt$

$= \int i \times v\,dt$

$= \int i \times L\frac{di}{dt} \times dt$

$E = \int i \times L\,di$

$E = \frac{1}{2}Li^2$



i: current through variable.

T-type element.

State variable: i ✓ Inductor

Observations: Inductor

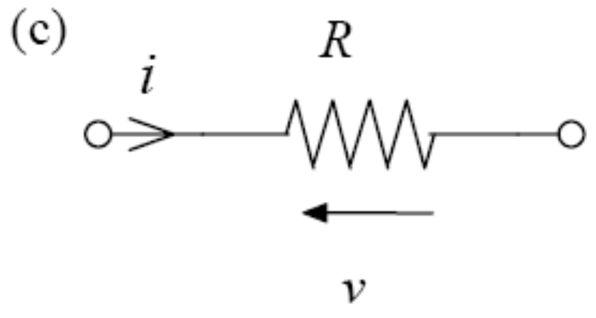
- Current (through variable) is state variable for an inductor → "T-Type Element".

- Current is a natural output variable and voltage is a natural input variable for an inductor.

- Current through an inductor cannot change instantaneously unless an infinite voltage is applied.

Electrical Element: Resistor (Dissipation)

Resistor Element (D-Type Element)



Constitutive Equation: $v = Ri$ (Ohm's law)
where R = resistance

D-type element

Observations:

1. This is an energy dissipating element **(D-Type Element)**

2. Either i or v may represent the state

3. No new state variable is defined by this element.

Electrical Components

Components	**Constitutive Equation**	**Energy Stored or Power Dissipated**
Capacitor	$i = C\frac{dv}{dt}$	$E = \frac{1}{2}Cv^2$
Inductor	$v = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$
Resistor	$v = iR$	$P = \frac{v^2}{R}$ or $P = I^2R$

Note:

- Voltage is a natural output variable and current is a natural input variable for a capacitor.
- Current is a natural output variable; voltage is a natural input variable and voltage is a natural state variable for an inductor.

Mechanical-Electrical Analogy

System Type System-Variables:	Mechanical	Electrical
Through-Variables	Force f	Current i
Across- Variables	Velocity v	Voltage v
System Parameters	m → mass k → sping b → damper	C $1/L$ $1/R$

groups categories {

Mechatronic systems { Mechanical / Electrical

Through / Across } ⇒ similarity

Thermal Elements

Variables: Across variable temperature (*T*) and through variable heat transfer rate (*Q*).

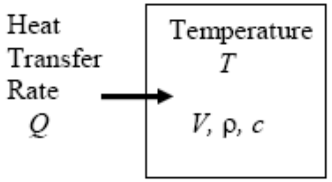
Thermal Capacitor (A-Type Element)

Consider control volume *V* of fluid with, density ρ, and specific heat *c*.

Constitutive Equation: Net heat transfer rate into the control volume $Q = \rho V c \frac{dT}{dt}$ è

$$C_t \frac{dT}{dt} = Q$$

$C_t = \rho v c$ = thermal capacitance of control volume



Observations:

Temperature *T* is state variable for thermal capacitor (from usual argument) è "**A-Type Element**"

Heat transfer rate *Q* is natural input and temperature *T* is natural output for this element

This is a storage element (stores thermal energy)

Note: There is no thermal "inductor" like storage element with state variable *Q* .

Thermal Elements (cont'd)

Thermal Resistance (D-Type Element)

Three basic processes of heat transfer è three different types of thermal resistance

Constitutive Relations

Conduction: $Q = \frac{kA}{\Delta x} T$

k = conductivity; A = area of cross section of the heat conduction element; Δx = length of heat conduction that has a temperature drop of T.

è Conductive resistance $R_k = \frac{\Delta x}{kA}$

Convection: $Q = h_c A T$

h_c = convection heat transfer coefficient; A = area of heat convection surface with temperature drop T

è Conductive resistance $R_c = \frac{1}{h_c A}$

Radiation: $Q = \sigma F_E F_A A(T_1^4 - T_2^4)$ è a nonlinear thermal resistor

σ = Stefan-Boltzman constant

F_E = effective emmisivity of the radiation source (of temperature T_1)

F_A = shape factor of the radiation receiver (of temperature T_2)

A = effective surface area of the receiver.

Fluid Elements

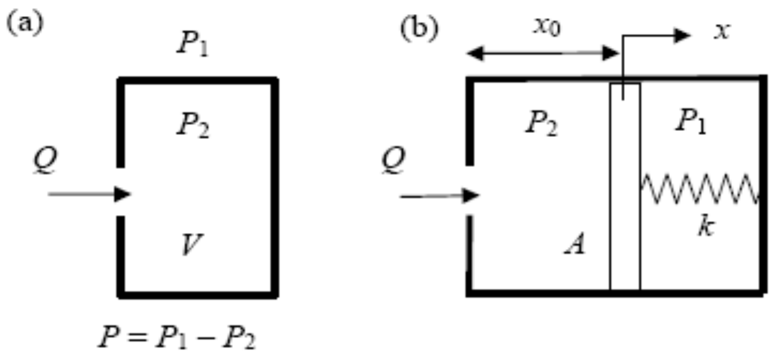
Variables: Pressure (across variable) P and volume flow rate (through variable) Q

Fluid Capacitor (A-Type Element)

Constitutive Equation: $C_f \frac{dP}{dt} = Q$

Note 1: Stores potential energy (a "fluid spring")

Note 2: Pressure (across variable) is state variable for fluid capacitor è "**A-Type Element**"



Three Types: Fluid compression; Flexible container; Gravity head

1a. For liquid control volume V of bulk modulus β: $C_{bulk} = \frac{V}{\beta}$

1b. For isothermal (constant temperature, slow-process) gas of volume V and pressure:

$$C_{comp} = \frac{V}{P}$$

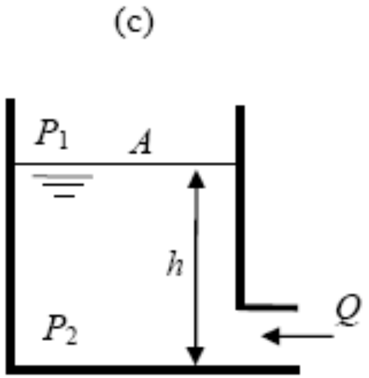
1. For adiabatic (zero heat transfer, fast-process) gas: $C_{comp} = \frac{V}{kP}$

$k = \frac{c_p}{c_v}$ = ratio of specific heats at constant pressure and constant volume

2. For incompressible fluid in a flexible vessel of area A and stiffness k: $C_{elastic} = \frac{A^2}{k}$

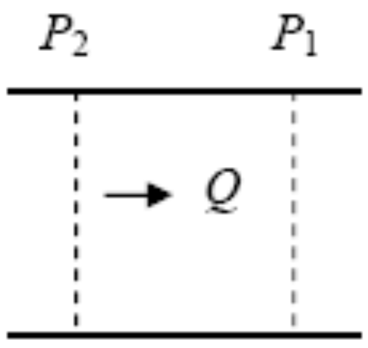
Note: For a fluid with bulk modulus, the equivalent capacitance $= C_{bulk} + C_{elastic}$.

3. For incompressible fluid column of area of cross-section A and density ρ: $C_{grav} = \frac{A}{\rho g}$



Fluid Elements (cont'd)

Fluid Inertor (T-Type Element)



Constitutive Equation: $I_f \frac{dQ}{dt} = P$

Note 1: Volume flow rate Q (through variable) is state variable for fluid inertor è "**T-type Element**"

Note 2: It stores kinetic energy, unlike the mechanical T-type element (spring), which stores potential energy.

With uniform velocity distribution across A over length segment Δx:

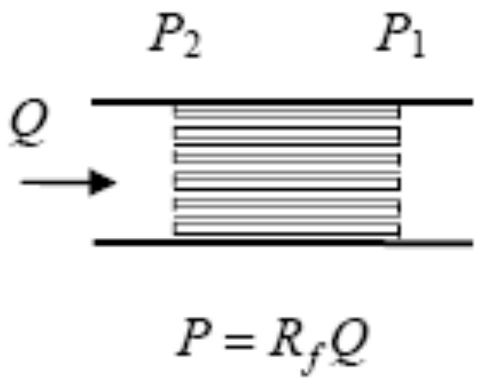
Fluid inertance $I_f = \rho \frac{\Delta x}{A}$

For a non-uniform velocity distribution:

Fluid inertance $I_f = \alpha\rho \frac{\Delta x}{A}$ (correction factor α)

For a pipe of circular cross-section with a parabolic velocity distribution, $\alpha = 2.0$

Fluid Resistor (D-Type Element)



Constitutive Equation (Linear): $P = R_f Q$

Constitutive Equation (Nonlinear): $P = K_R Q^n$

(K_R and n are parameters of nonlinearity)

For Viscous Flow Through a Uniform Pipe:

(a) With circular cross-section of diameter d: $R_f = 128\,\mu \dfrac{\Delta x}{\pi d^4}$

(b) With rectangular cross-section of height b << width w: $R_f = 12\mu \dfrac{\Delta x}{w b^3}$

Note: μ = absolute viscosity (or, dynamic viscosity); υ = kinematic viscosity with $\mu = \upsilon\rho$

Analogies and Constitutive Relations

System Type	Constitutive Relation for		
	Energy Storage Elements		Energy Dissipating Elements
	A-Type (Across) Element	T-Type (Through) Element	D-Type (Dissipative) Element
Translatory-Mechanical v = velocity f = force	Mass (Newton's 2nd Law) m = mass	Spring (Hooke's Law) k = stiffness	Viscous Damper b = damping constant
Electrical v = voltage i = current	Capacitor C = capacitance	Inductor L = inductance	Resistor R = resistance
Thermal T = temperature difference Q = heat transfer rate	Thermal Capacitor C_t = thermal capacitance	None	Thermal Resistor R_t = thermal resistance
Fluid P = pressure difference Q = volume flow rate	Fluid Capacitor C_f = fluid capacitance	Fluid Inertor I_f = inertance	Fluid Resistor R_f = fluid resistance

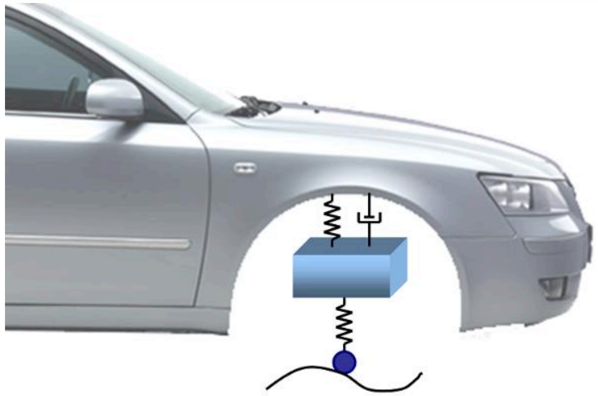
elements

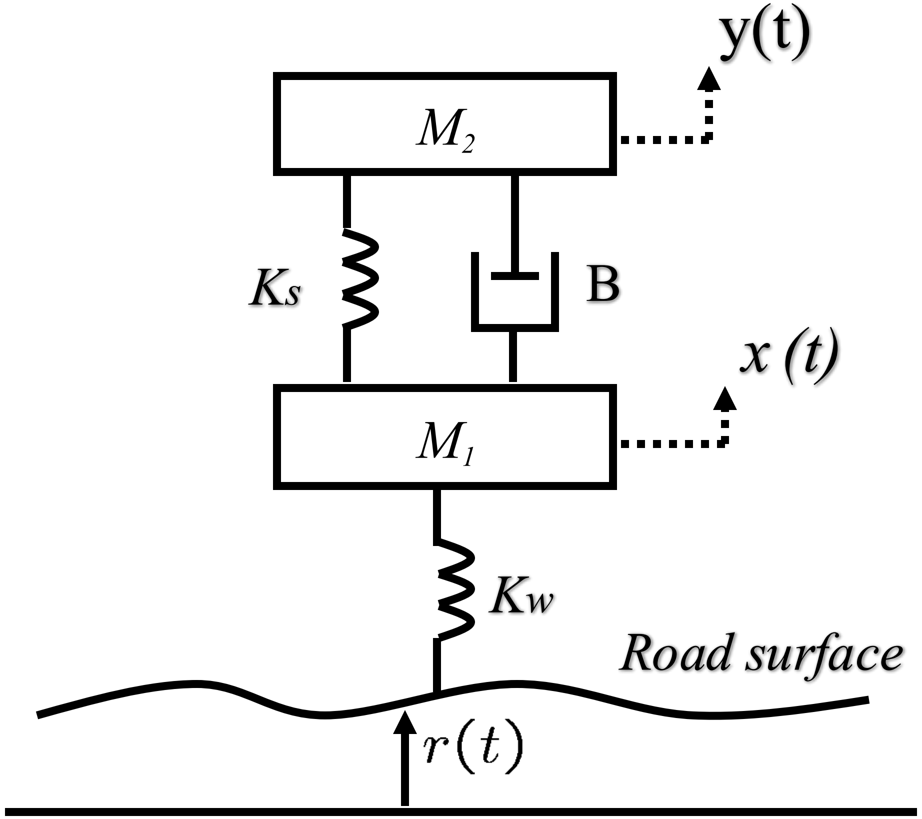
Through and Across Variables

System Type	Through Variable	Across Variable
Hydraulic/Pneumatic	Flow Rate	Pressure
Electrical	Current	Voltage
Mechanical	Force	Velocity
Thermal	Heat Transfer	Temperature

Building Up Mechanical Systems

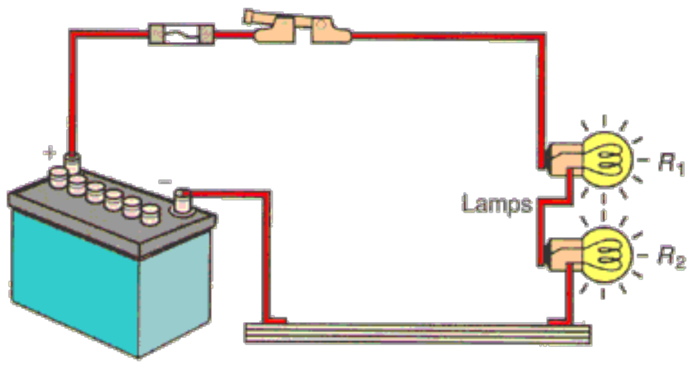
Suspension of a car

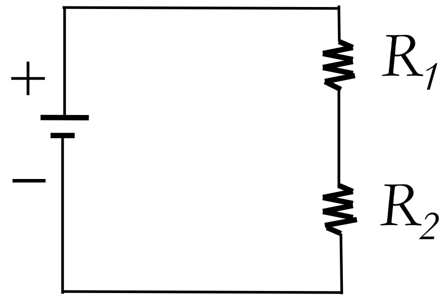


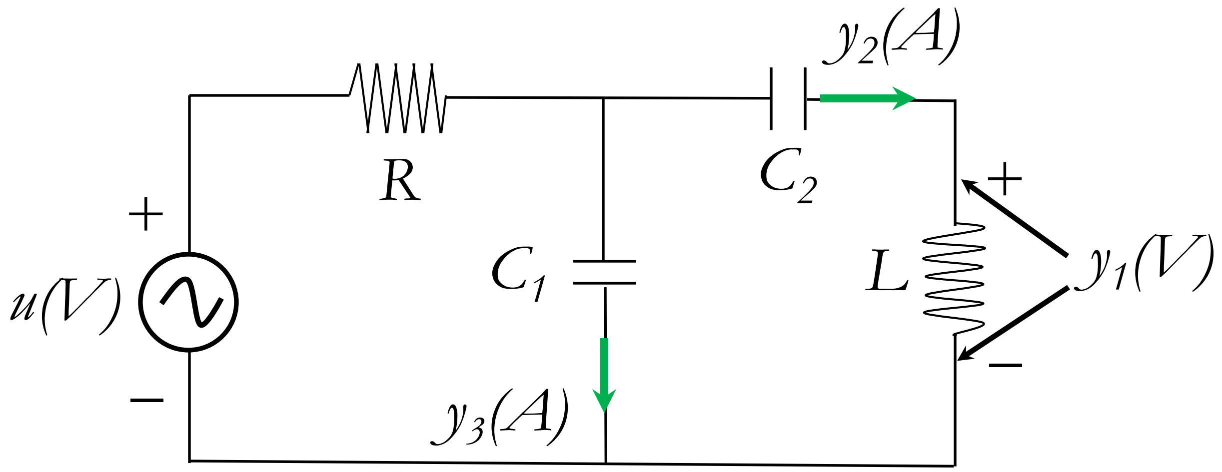


Building Up Electrical Systems

Electrical Circuit

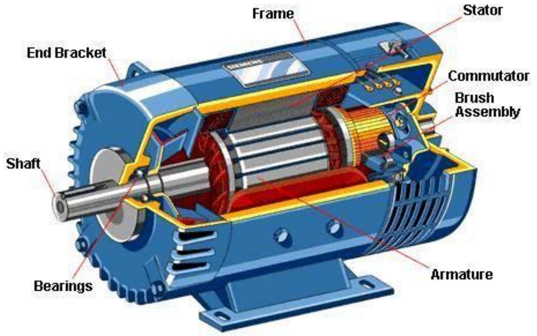


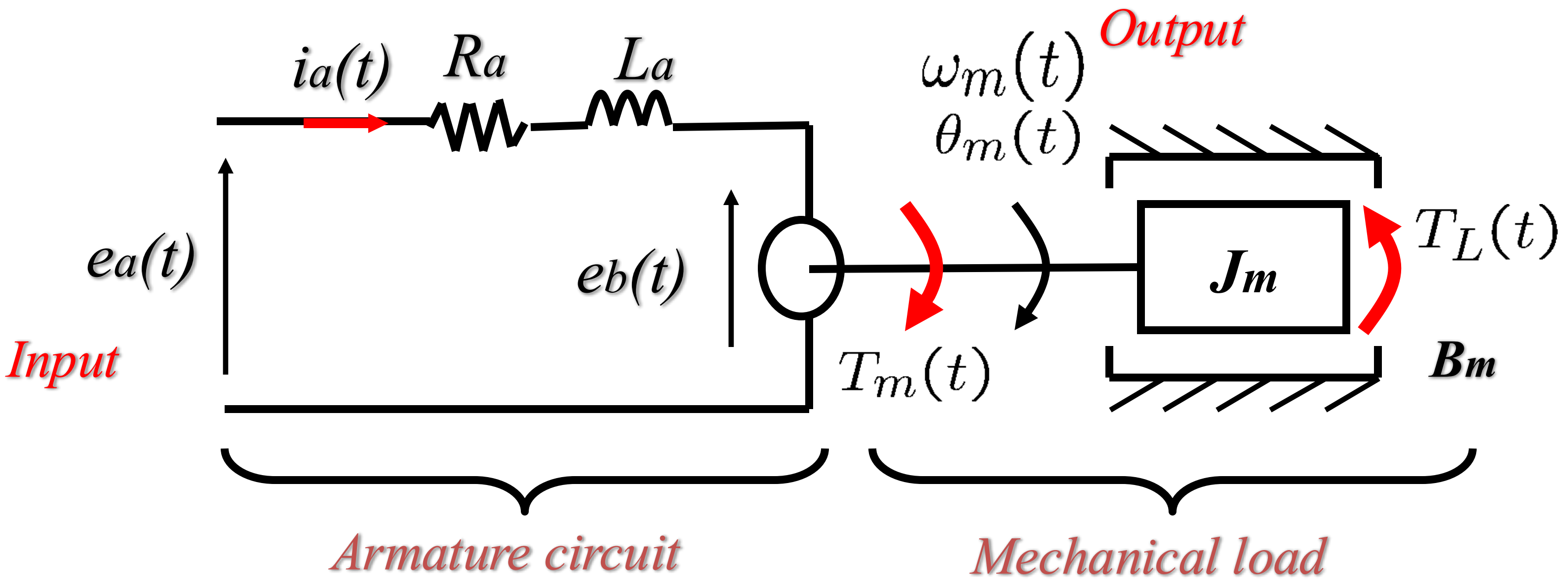




Building Up Mechatronic Systems

DC Motor (will discuss it in detail in later chapter)





The End!!